High resolution objective analysis of historical SSTs: small-scale, long-term, and sea-ice aspects

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Paradigm of the least squares optimal estimation

To estimate:
\[ T = T_{\text{total}} - T_{\text{clim}}, \quad \langle T \rangle = 0. \]

Input:
\[ T^o = HT + e^o, \quad \langle e^o \rangle = 0. \]

A priori information:
\[ \langle e^o e^{oT} \rangle = R, \]
\[ \langle TT^T \rangle = C. \]

An educative example: reconstructing SSTs from COADS data for the NCEP OI period: Nov 1981 - 2001. No space reduction!
Actual error (cf NCEP OI)

Theoretical error

Available COADS  Missing COADS
RMS(COADS-NCEPOI)

Theoretical obs error

SSV/Nobs

SSV from COADS

SSV from L.Kent-RTG+NCEPOI

Needed SSV! ;-(
Small-scale variability in the signal and data error can be separated from the large-scale signal variability that presumably can be estimated in a more robust way...

\[
\begin{align*}
\mathcal{T} &= (H^T R^{-1} H + C^{-1})^{-1} H^T R^{-1} T^o = C H^T (R + H C H^T)^{-1} T^o = \\
&= E \Lambda E^T H^T (H E \Lambda E^T H^T + H E' \Lambda E'^T H^T + R)^{-1} T^o + \\
&\quad E' \Lambda E'^T H^T (H E \Lambda E^T H^T + H E' \Lambda E'^T H^T + R)^{-1} T^o = \\
&= E \alpha + C' H^T (H E \Lambda E^T H^T + H C' H^T + R)^{-1} T^o = \\
&= E \alpha + C' H^T (C' H^T + R)^{-1} \Delta T^o = E \alpha + \Delta T
\end{align*}
\]

Introduce \( \Delta T^o = T^o - H E \alpha \) and \( \Delta T = T - H E \alpha \).

OI problem for small scale corrections:

\[
H \Delta T = \Delta T^o + \varepsilon^o,
\]

\[
\langle \Delta T \Delta T^T \rangle = C', \quad \langle \varepsilon^o \varepsilon^{oT} \rangle = R
\]

Solution:

\[
\Delta T = C' H^T (C' H^T + R)^{-1} \Delta T^o
\]

with error

\[
P = C' - C' H^T (C' H^T + R)^{-1} H C'.
\]
Residual data consists of error, physical small-scale variability and long-term variability (trends)
Reestimation of sliding climatology and window covariance is possible at this stage!
Linear trends 1900-1991
Linear trends 1951-1991

K98

HadISST1.1

OS

OS+res_OI
Linear trends 1900-1950
CONCLUSIONS

1. In order to obtain reliable error estimate we need to have better knowledge of small-scale variability in signal and data error.

2. Scale separation approach allows to work towards conceptually uniform globally-complete high-resolution objective analyses of SST according to the following scheme:
   (a) start from the reduced-space analysis (with an assumption of stationary mean and covariance);
   (b) small-scale analysis of observational residuals;
   (c) recomputing non-stationary mean and covariance;
   (d) adding high-resolution corrections, globalization patches, and sea-ice analyzed fields by the same scheme: large-scale prediction from the SST fields + local-scale corrections.