Towards a Dynamically constrained Analysis of Sea Level Pressure and Winds

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1. Introduction

The collection of marine observations in COADS has been an important resource for the study of climate variability over the globe. In their most frequently used form, these data are averaged by month and over 2°x2° ocean areas. This representing is useful for the study of climate variability at the ocean atmosphere interface. It also minimizes random errors due to instrumental and measurement errors. However, averaging together observations that are non-uniformly distributed in space and time may lead to sampling errors with respect to the true monthly mean values. These errors may also be considered random. Systematic errors occur due to the systematic geographical biases in sampling, such as with vessels adhering to common routes. Other errors result from averaging together observations made with different instruments or techniques. All these errors obscure the signal of climate variability and their effect should be minimized before the data are used for diagnostic studies. It appears plausible that by modeling the errors, and using physical relationships to constrain the data within the context of a statistical analysis, we can improve the quality of the monthly mean estimates in COADS. This approach could also help fill in gaps in the record and yield a uniform two-dimensional representation of the state variables. This paper describes the development of a statistical analysis, aimed at removing random errors from wind and sea level pressure data by using a variational technique (e.g., Sasaki, 1970) under dynamical constraints.

The present analysis is motivated by a recent study by Zebiak (1990) concerning the wind field over the equatorial Pacific. Zebiak used a uniformly gridded representation of the windstress field, prepared at the Florida State University (FSU, Goldenberg and O'Brien, 1981), to examine the vorticity budget, and the implied structure of boundary layer forcing near the equator. The gridded data exhibit a large imbalance in the vorticity budget, randomly distributed over the area considered. This imbalance is most likely due to random data errors. By constraining the data to agree with a steady-state, linear momentum balance, Zebiak was able to correct the wind field within the uncertainty of the observations, and improve the representation of the monthly mean wind anomalies. This approach appears to provide a simple and useful method to constrain the monthly box averages in COADS. We attempted to generalize the analysis to incorporate both wind and pressure data, and relax the constraint imposed in Zebiak (1990), by formulating it in terms of a variational principle. In the development stage of this procedure, we chose to use regularly gridded data to obtain a simplified setting, and a mean to compare the results with those of Zebiak (1990). We therefore made use of the FSU data and a regularly gridded monthly sea level pressure field provided by the National Meteorological Center (NMC). Our goal is to apply the analysis to the COADS data once its results are evaluated. The following sections describe the data (Section 2), the analysis procedure (Section 3) and the results of this preliminary study (Section 4).

2. Data

Monthly mean surface winds were calculated from the monthly FSU pseudo-stress field for the period 1979-88. These data are available on a 2° x 2° grid of the equatorial Pacific from 29°S to 29° N and from 124°E to 70°W. The data were interpolated to a 4° latitude by 6° longitude grid using a Gaussian smoother to simplify the calculations and remove some of the small scale features. The climatological mean values for each calendar month were removed from the data to create monthly anomalies.

The sea level pressure field for the period 1979-88 was obtained from NMC on a 5° latitude by 10° longitude grid. The data were transformed onto a 4° x 6° grid to be used with the winds in the analysis. Monthly anomalies were calculated with respect to the long-term average for each month.

3. ANALYSIS

a. Variational analysis with strong constraints

The analysis procedure adopted by Zebiak entails both dynamical and statistical constraints. The dynamical constraints impose a steady-state, linear momentum balance in the presence of a linear friction law. These constraints can be written as follows:

$$\begin{aligned} \varepsilon u - fv &= -p_x/\rho \\ \varepsilon v + fu &= -p_y/\rho \end{aligned} \qquad (1)$$

Here u and v are the zonal and meridional wind components, p is pressure, and ρ is the density of air (constant). Friction is represented by a constant ε equal to the inverse dissipation time (taken as 1 day). Using these equations one can apply the trapezoidal rule to integrate the wind field and obtain a consistent pressure. If the linear momentum balance holds, the pressure increment

$\widehat{q^x_{i,j}}$, between gridpoint (i, j), and point $(i+ l, j)$ is given by:

$$\widehat{q^x_{i,j}} = -\rho[\varepsilon(u_{i,j} + u_{i+1,j}) - f_{i,j}(v_{i,j} + v_{i+1,j})]\frac{\Delta x}{2} \qquad (2)$$

where Δx is the grid spacing in the zonal direction. Similarly, the pressure increment in the y direction is given by:

$$\widehat{q^y_{i,j}} = -\rho[\varepsilon(v_{i,j} + v_{i,j+1}) + (f_{i,j}u_{i,j} + f_{i,j+1}u_{i,j+1})]\frac{\Delta y}{2} \qquad (3)$$

where Δy is the grid spacing in the meridional direction. It should be realized, however, that the pressure field can not be uniquely determined in this manner because of data errors, and the fact that the dynamical balance in (1) is only an approximation. The derived pressure field will therefore depend on the path of integration.

To obtain an optimal result in his study, Zebiak first integrated the equations along a latitude line, beginning from an arbitrary pressure value. From this latitude, the pressure was then integrated northward and southward to determine the pressure in the entire domain. This procedure was repeated for all other latitudes on the grid, and the final pressure field determined from a weighted average of all the resulting pressure fields. The free constant of the integration was determined by requiring that the domain averaged pressure vanishes.

While this procedure appears to be arbitrary it can be shown that it represents a simple variational principle. This principle can be expressed as the least squares fit between the analyzed pressure increment between two gridpoints in the zonal direction, and the sum of frictional and coriolis terms denoted by $\widehat{q^x}$, in (2). Formally the problem becomes the minimization of the cost function:

$$S = \frac{1}{2}\sum_{i=1}^{I-1}\sum_{j=1}^{J}\left[\left(P_{i+1,j} - P_{i,j}\right) - \widehat{q^{x}_{i,j}}\right]^2 \omega_j \qquad (4)$$

with respect to the unknown, analyzed pressure values P_{ij}, under the constraint that the pressure increment in the meridional direction always satisfies the linear, steady-state momentum balance implied by the data, i.e.,

$$P_{i,j+1} - P_{i,j} = \widehat{q^{y}_{i,j}} \qquad (5)$$

Here the $\hat{q}$ terms are calculated from the observed wind components according to (2) and (3), and ω_j is a latitude dependent weight. I and J are the maximum number of gridpoints along the latitude and longitude lines, respectively. Pressure data are not used in the analysis, but a pressure field is calculated and used to derive an analyzed wind field that is dynamically constrained to satisfy (1).

The constraint expressed in (5) is referred to as a strong constraint (e.g., Sasaki, 1970). In generalizing the algorithm used by Zebiak (1990), we sought to relax the assumption that the pressure gradient in the meridional direction satisfies the balance (3) exactly. This can be achieved by formulating the problem as a weakly constrained variational principle. In the present study two such formulations were used. In the first one, wind data only were used and the pressure field was not explicitly derived. In the other formulation, both pressure and wind data were used and analyzed.

b. Weakly const.-ained analysis of winds only

To formulate the previous analysis as a weakly constrained problem we note that the requirement that the pressure field is uniquely determined can be written as a requirement that the line integral of the pressure difference between the grid box corners vanishes, i.e.,

$$(P_{i+1,j} - P_{i,j}) + (P_{i+1,j+1} - P_{i+1,j}) - (P_{i+1,j+1} - P_{i,j+1}) - (P_{i,j+1} - P_{i,j}) = 0 \qquad (6)$$

where the upper case letters P denotes the unknown, analyzed pressure. The requirement that the linear momentum balance holds can be introduced by replacing the pressure differences in (6) by their linear equivalents according to (2) and (3), i.e.,

$$M_{i,j} \equiv \widehat{Q^x_{i,j}} + \widehat{Q^y_{i+1,j}} - \widehat{Q^x_{i,j+1}} - \widehat{Q^y_{i,j}} = 0 \qquad (7)$$

where upper case letters $\hat{Q}$ were used to denote analyzed values. The equality holds only if the stationary, linear balance is strictly imposed. Thus the requirement $M = 0$ expresses the dynamical constraints on the analyzed wind field.

Using (7) and the additional requirement that the analysis (denoted by upper case letters) stays "close" to the observations (lower case letters), a new cost function is defined:

$$S = \frac{1}{2}\left\{ \sum_{i=1}^{I-1}\sum_{j=1}^{J-1} \mu_{i,j}\left(M^2_{i,j}\right) + \sum_{i=1}^{I-1}\sum_{j=1}^{J} \alpha_{i,j}\left(\widehat{Q^x_{i,j}} - \widehat{q^x_{i,j}}\right)^2 + \right.$$

$$\left. \sum_{i=1}^{I}\sum_{j=1}^{J-1} \beta_{i,j}\left(\widehat{Q^y_{i,j}} - \widehat{q^y_{i,j}}\right)^2 \right\} \qquad (8)$$

Minimizing (8) with respect to the variables $\widehat{Q}$ results in the analyzed field. The wind components can be calculated using the analyzed values $\widehat{Q^x}$ and $\widehat{Q^y}$, and the linear relationships (2) and (3). What makes this a weakly constrained problem is the fact that the parameters α, β, and μ are set before the minimization is performed (Sasaki, 1970). The guidelines for the choice of these parameters are that they should be related to the inverse error variances of the term they multiply (e.g., Wunsch, 1989). In this way the terms that are associated with relatively small errors constrain the minimization more than the terms that have larger errors. Note that in this case we do not solve for the pressure field explicitly.

c. Weakly constrained analysis of winds and pressure.

To include the pressure field in the analysis we express the dynamical constraints as:

$$
\begin{aligned}
P_{i+1,\,j} - P_{i,\,j} &= \widehat{Q}^{x}_{i,\,j} \\
P_{i,\,j+1} - P_{i,\,j} &= \widehat{Q}^{y}_{i,\,j}
\end{aligned}
\qquad (9)
$$

This enabled us to write the variational principle as:

$$
S = \frac{1}{2}\sum_{i=1}^{I-1}\sum_{j=1}^{J}\left[2_{i,\,j}\left(P_{i+1,\,j} - P_{i,\,j} - \widehat{Q}^{x}_{i,\,j}\right)^{2} + \alpha_{a,\,j}\left(\widehat{Q}^{x}_{i,\,j} - \widehat{q}^{x}_{i,\,j}\right)^{2} + \right]
$$

$$
\frac{1}{2}\sum_{i=1}^{I}\sum_{j=1}^{J-1}\left[\delta_{i,\,j}\left(P_{i,\,j+1} - P_{i,\,j} - \widehat{Q}^{y}_{i,\,j}\right)^{2} + \beta_{i,\,j}\left(\widehat{Q}^{y}_{i,\,j} + \widehat{q}^{y}_{i,\,j}\right)^{2} + \right]
$$

$$
\frac{1}{2}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\pi_{i,\,j}(P_{i+1,\,j} - p_{i,\,j})^{2}\right] \qquad (10)
$$

Here again, upper case letters denote the unknown analyzed values and the lower case ones are calculated or taken from the observations. The pressure field is explicitly derived after minimizing the expression with respect to all unknown variables. The new parameters γ, δ, and π are the estimates of the inverse error variances of the terms they respectively multiply.

4. Results

We performed three different analyses of the equatorial wind field. The minimization of the cost functions (4), (8), and (10) was performed by using a conjugate-gradient routine from IMSL.

As noted above, it is important to choose the parameters in the different cost functions so that they reflect the error variances of their respective terms. In this way, terms that are expected to have a large error are weighted less, and therefore constrain the results less than terms that are expected to be more accurate. We calculated the errors in the different terms starting from the assumption that the typical monthly wind anomaly is 1 ms^{-1} and its error is 0.5 ms^{-1}. The pressure error was taken as 30 Pascal. Using the relations (2), (3) and these values, we derived the errors in $\widehat{q}^{x}$ and $\widehat{q}^{y}$. The parameters α and β are the squared inverses of these errors. Because of the coriolis factor, these parameters depend on latitude. To determine the error in the dynamical constraints, we assumed that the imbalance in (1) is related mainly to the frictional term. Taking the uncertainty in ε to be 50%, we used simple error analysis to determine the expected error in the models (7) and (9), and subsequently the parameters γ, δ, and μ. While these choices seem arbitrary, we note that they can be checked against the result of the analysis. An analysis that is not consistent with the assumptions made previously suggests that these assumptions should be modified and the minimization repeated until consistent results are achieved. Note also that the variational principle implicitly assumes that the errors are random and do not have a coherent spatial structure.

The results of all three analyses were qualitatively similar when compared to the input data. As a point of reference, Fig. la, b show the rms monthly zonal and meridional wind anomalies, u, v, respectively. These figures depict the minima associated with the steady trades, and the maxima associated with interannual and seasonal variability along the equator, in particular just west of the date line. When the data are analyzed, the magnitude of the rms values is reduced but what is more important, the mean structure of the monthly anomalies becomes better defined. This is shown here by displaying the results of the third analysis (weak constraints applied to wind and pressure) in Figure 2. The rms differences between the “observed” winds (i.e., those derived from the FSU data), and the winds analyzed using the strongly constrained principle (4), (5) are shown in Fig. 3. Similarly, Figure 4 is for the weakly constrained analysis using both pressure and winds according to (10). In both cases the corrections imposed by the analysis are about 50% of the local monthly wind anomaly (compare to Fig. 1), consistent with our assumption regarding the observational errors. The correction to the pressure field in the weakly constrained analysis (Fig. 5) ranges from 25 to 50 pascal, consistent with the assumed pressure error. Note however that there is structure in these “error” terms that may be related to either the dynamical model used, or to systematic errors in the observations due to sampling disparity.

The divergence component of the FSU winds is quite noisy and shows little geographical structure, as can be seen by plotting the rms value of divergence (Fig. 6a). The analyses filter out the spurious divergence and depict the expected maximum along the equator, e.g., as in the case of the weakly constrained analysis (Fig. 6b). The vorticity field (not shown) is modified less by the analysis.

5. Summary

The results of the preliminary study presented above are encouraging. It was demonstrated that using a variational principle constrained by an approximate dynamical relationship between pressure and wind can significantly reduce random errors in the data. These results were obtained using uniformly gridded data in what constitute relatively smooth products. It is our goal to generalize the analysis further so that it could be applied to the “trimmed” version of COADS. This generalization will involve accounting for additional constraints of a statistical nature (i.e., climatology and the covariance matrix), to allow the filling of gaps in the record.

Acknowledgments:

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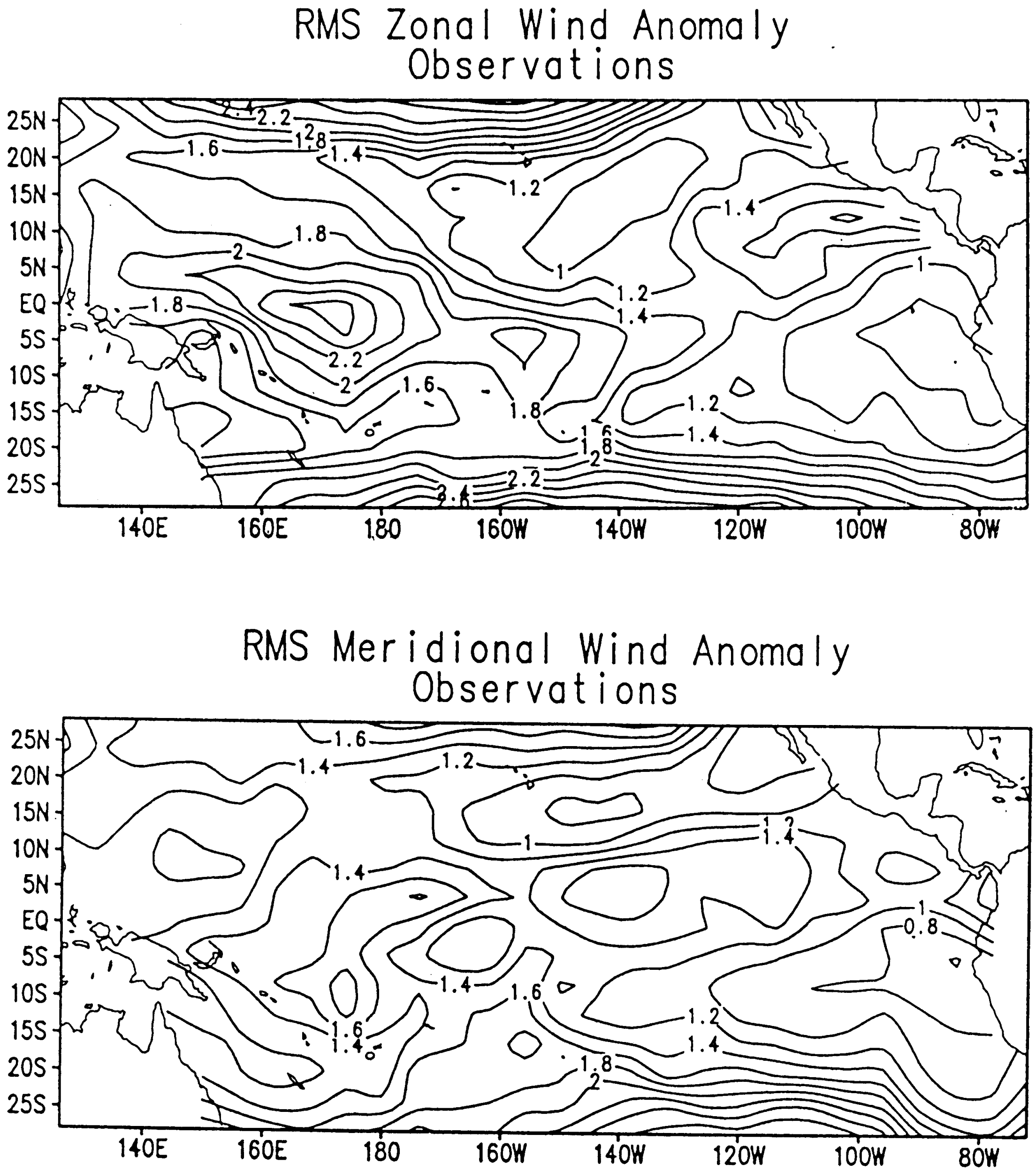


Figure 1. The rms monthly zonal wind anomaly (a) and meridional anomaly (b), calculated from FSU data. Contour interval is 0.2 ms^{-1}.

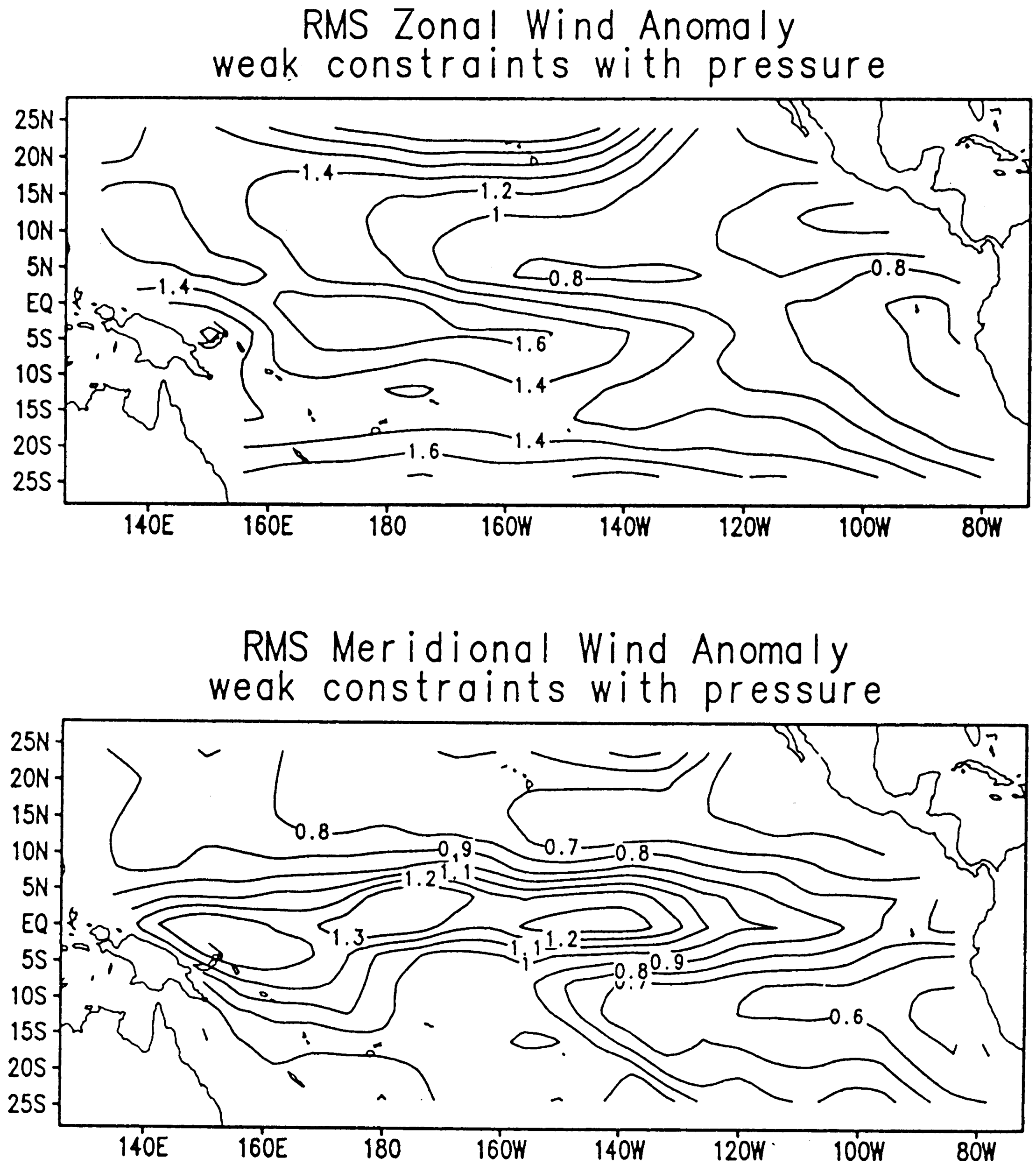


Figure 2. Same as Figure 1, but calculated from the weakly constrained analysis of pressure and winds.

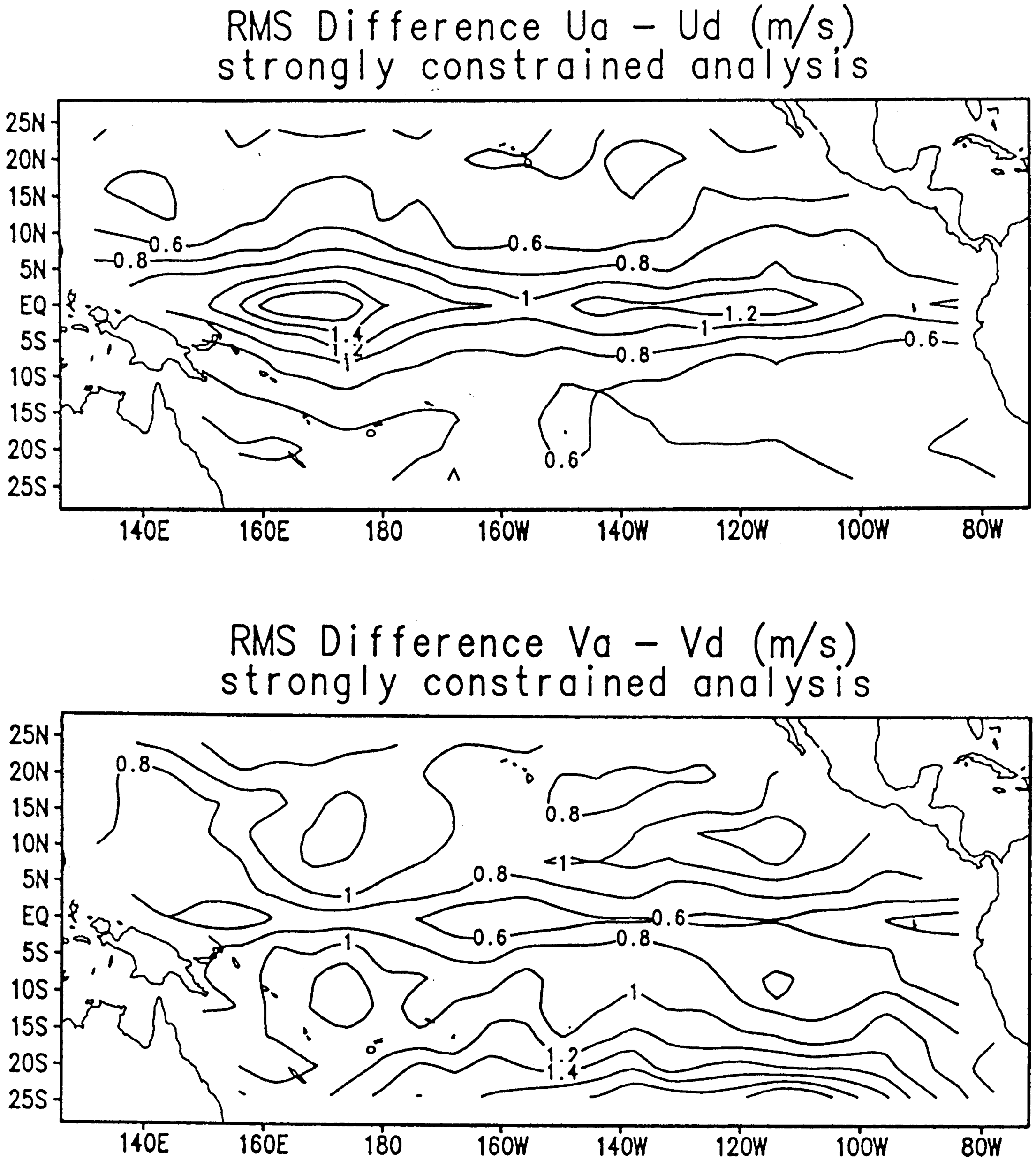
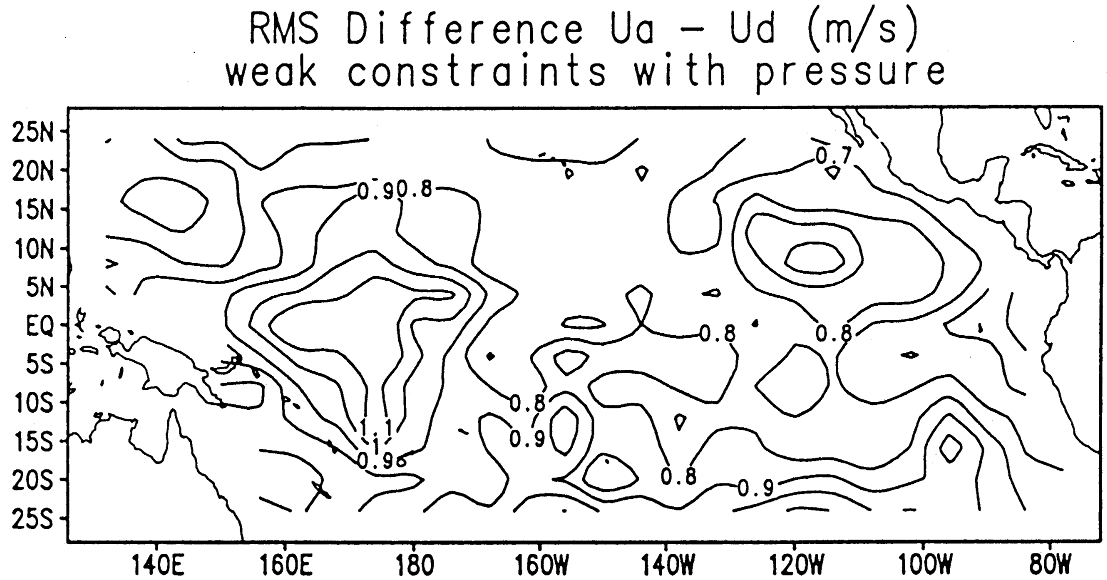


Figure 3. The rms difference between the strongly constrained analysis and the FSU data for the zonal wind component (a) and the meridional component (b). Contour interval is 0.2 ms^{-1}.



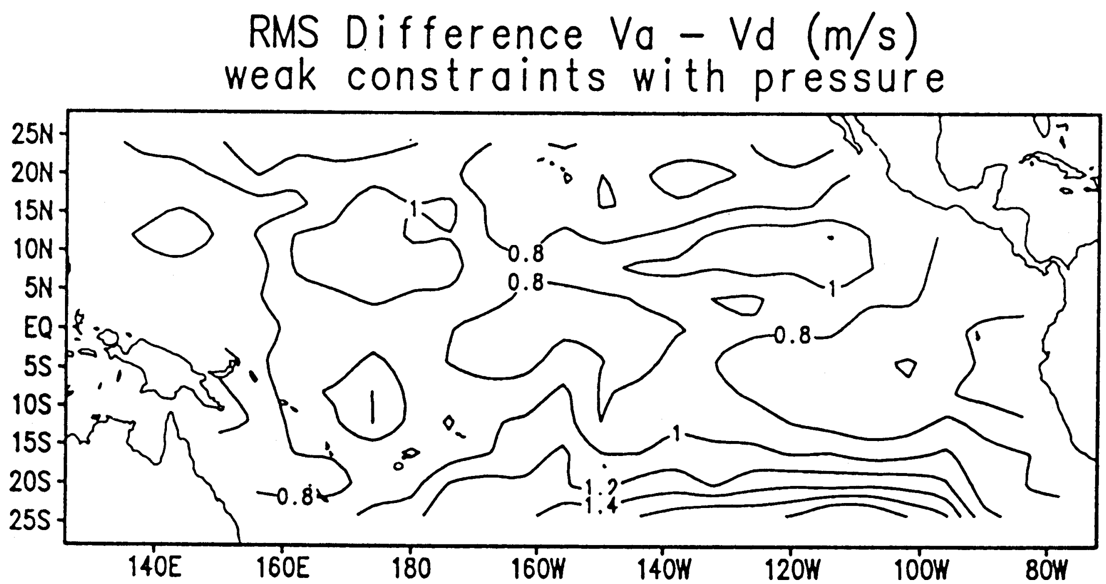


Figure 4. Same as in Figure 3, but for the weakly constrained analysis of winds and pressure. Contour interval is 0.1 ms^{-1} in (a) and 0.2 ms^{-1} in (b).

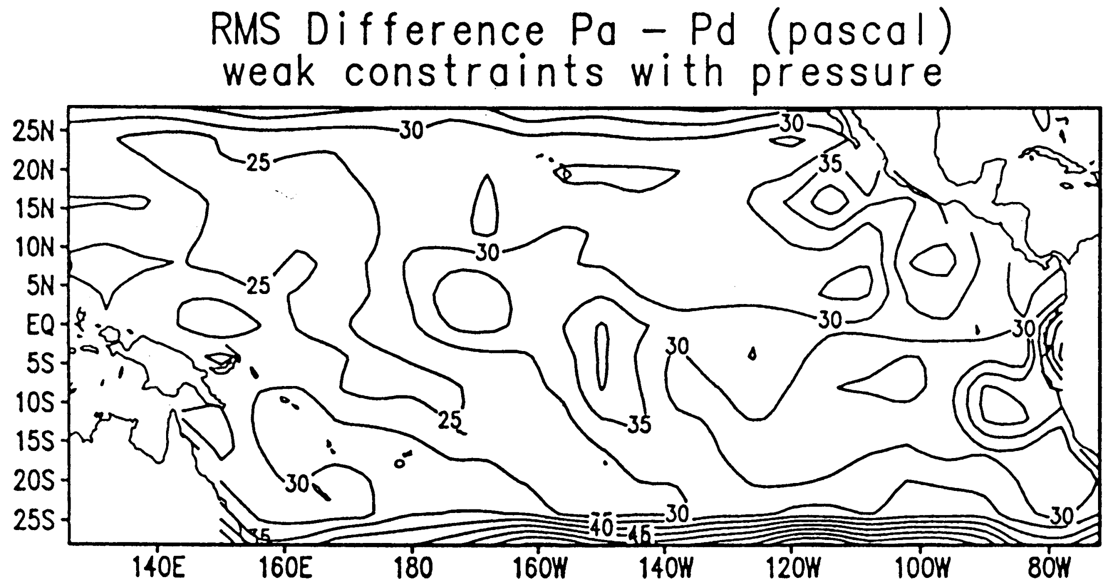


Figure 5. The rms difference between the analyzed sea level pressure field and pressure from NMC data. Contour interval is 5 pascal

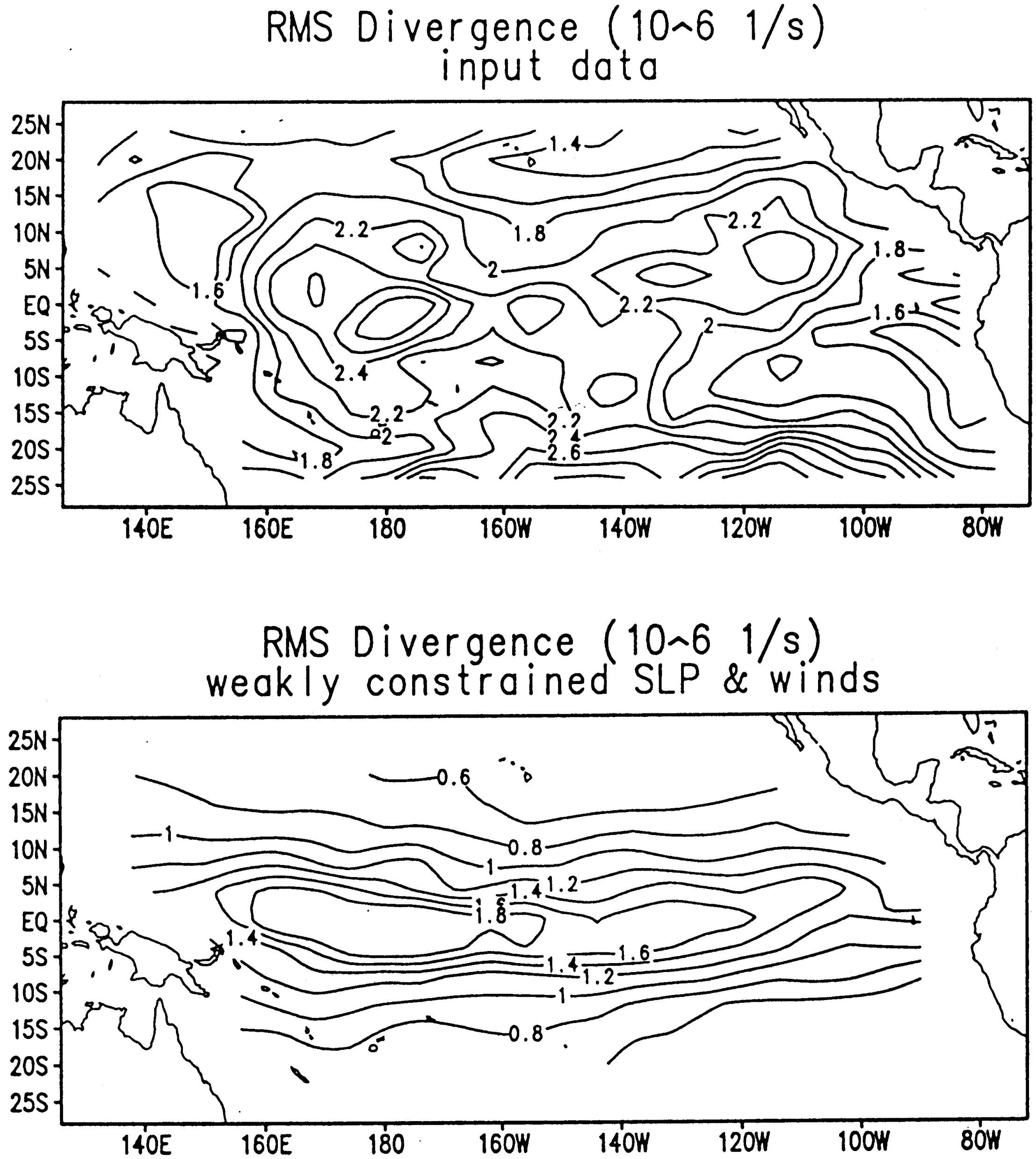


Figure 6. The rms divergence calculated from the FSU data (a) and from the weakly constrained analysis pressure and winds (b). Contour interval is 0.2 $10^{-6}s^{-1}$.